The Photon and the Second Order Correlation Function: Experimental Demonstration with the quED

1 Introduction

In this session, we would like you to get to know our experimental setup, how we produce and detect photon pairs and how we can demonstrate a fundamental principle of quantum physics, namely the quantisation aspect. We will use a very experimental approach, we will not strictly derive anything, but try to motivate some issues. Please take this into consideration.

1.1 SPDC Setup

Before we start with the experiments themselves, we will give you a brief overview of the equipment we are using. This qutools product is called *quED* (Entanglement Demonstrator) and at its core there is a so-called Spontaneous Parametric Down-Conversion (SPDC) source. The most important parts of the SPDC source are:

- Pump laser
- Non-linear β-Barium Borate (BBO) crystal

The pump laser (wavelength $\lambda_p \approx 405$ nm) illuminates the non-linear crystal (see Fig. 1). Because of the non-linearity, sum and difference frequencies can be excited. Energy conservation is still fulfilled, so $\omega_p = \omega_s + \omega_i$. The subscripts s and i mean "signal" and "idler", but in our case it does not matter which one is which; if we do everything correctly, they are indistinguishable, but that is not important now, either. In the degenerate case, which should be the one we are producing, $\omega_s = \omega_i = \frac{\omega_p}{2} \approx 810$ nm. Since "signal" and

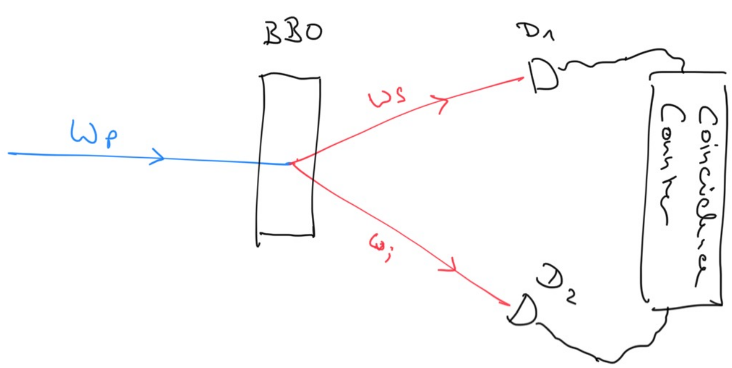

Figure 1: The general situation of the SPDCSpontaneous Parametric Down-Conversion (SPDC) process. Pump light of wavelength ω_p gets "down-converted" by use of the BBO crystal into signal (ω_s) and idler (ω_i) beams. This image depicts the non-collinear case, where signal and idler beam do not spatially overlap with the pump beam. D_1 and D_2 mark single photon detectors in each arm.

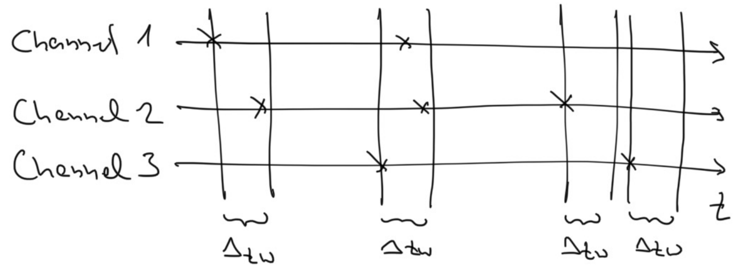

Figure 2: Coincidence detection/evaluation. The first two (leftmost) events are detected within the coincidence time window Δ_{tw}. They are counted as a (2-fold) coincidence. The next three events all happen within a coincidence time window, so they are counted as a 3-fold coincidence. The next two events are so-called "singles", since they do not have a partner event within Δ_{tw}.

"idler" are not really distinguishable here, we will mostly refer to the signal and idler paths as two "arms" of the down conversion source and just enumerate them.

If you would like to know more about the SPDC process, please refer to e.g. [1] or for a more theoretical appraisal e.g. [2]. A similar source has been described in [3].

1.2 Detecting the SPDC Output

Important components for the detection and analysis of the detection events:

- Single photon detectors (usually at least 2)
- Coincidence counter

At the end of each arm or path, we usually place single photon detectors, tagged D_1 and D_2 in Fig. 1. They convert very low light levels into easily measurable electronic signals. These are evaluated by what we refer to as a "coincidence counter". This is a device that counts the detector output signals. For each input channel singly, but also events that happen more or less simultaneously. If n detector events (from different detectors[1]) happen within a time window of width Δ_{tw} (called "coincidence time window"), we call that an n-fold coincidence. 3-fold coincidences are also counted as 3 2-fold coincidences and 2-fold coincidences are also counted as single counts (number of 3-fold coincidences $\leq$ number of 2-fold coincidences $\leq$ number of singles).

We are using so-called Single Photon Avalanche Detectors (SPADs) made from Silicon. They are sometimes also called "Avalanche Photon Detectors (APDs)". Their inputs are connected to optical fibres, so we have to get the output of the SPDC source into these fibres. The SPADs can detect single photons[2] with a wavelength dependent efficiency (in our case roughly 30 % at 810 nm). Our coincidence time window has a width of 30 ns with the standard settings in place.

2 Existence of the Photon/Single Photons/Particle Character of the Photon

Now that we know what our setup in general looks like, let us see whether we can demonstrate that it produces these "photons". We want to show these have particle character. What does that mean? (Elementary) particles cannot be divided it into smaller parts.

[1] Most of these detectors cannot distinguish between one and more photons, they will simply output one pulse with a fixed height and they need some time to recover from a detection event, the so-called "dead time".

[2] The existence of a "Photon" as a quantum of light is just a hypothesis at the moment.

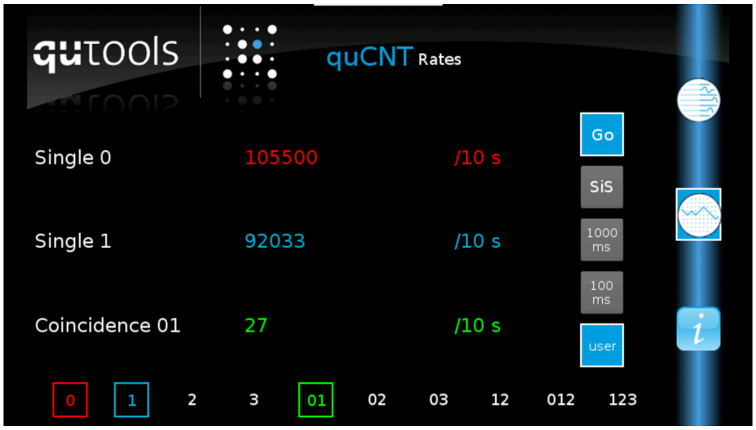

Figure 3: Visual output of the coincidence count rates: Singles of channels 0 and 1 and coincidences between channels 0 and 1 (“01”).

From experience we know, that if we have a partially reflecting surface, ideally a 50:50 beam splitter, and shine light onto it, a part of the intensity will be transmitted and the rest (neglecting losses) will be reflected (see left part of Fig. 4).

2.1 Single Photon Source

If we had a single, indivisible “light particle”, which we will call “photon” from now on, this could only be either transmitted or reflected (center or right part of Fig. 4). So, there would not be any coincidences between the two detectors in this setup. Let’s do the experiment!

We set up the quED and connect a (fibre based) Beam Splitter (BS) to one of the arms of the SPDC source, with a SPAD at each output port of the BS. We ignore the second arm of the source completely for the moment. Fig. 5 shows the setup.

Now, we want to look at 2-fold coincidences between detector 1 and 2. The result of the experiment has already been shown in Fig. 3. The coincidence counter[3] has counted for 10 s and the number of coincidences is not 0 as we might have expected. In an ideal experiment (this is an antithesis in itself) with an ideal single photon source and ideal detectors etc., this would be correct[4]. But, our setup is far from perfect. If

[3]Our counter starts counting the channel numbers at 0, so please always add one in your head.

[4]Actually, even then this would not be quite sufficient. An easy way to get this value to 0 is switching off both detectors. So you would need to add the condition that the single count rates are not 0.

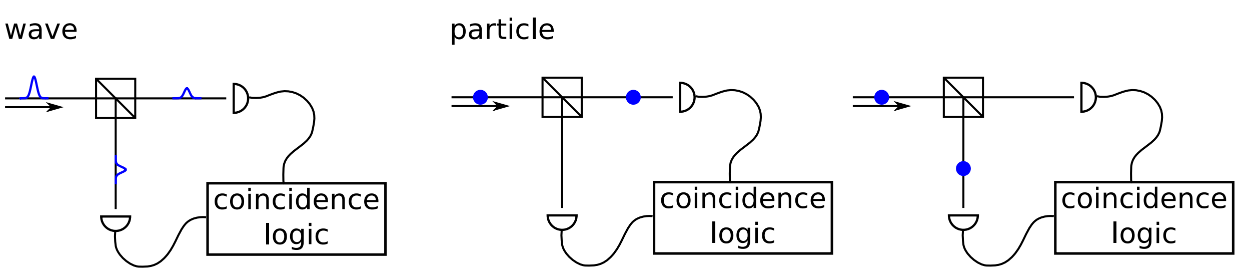

Figure 4: In the wave picture, the intensity of the incoming wave should be distributed across the outputs. There is no reason to believe that this cannot be done anymore, when the light levels get very dim (left part). When light is imagined as consisting of elementary particles, that cannot be divided, a single one of these particles can only be transmitted or reflected.

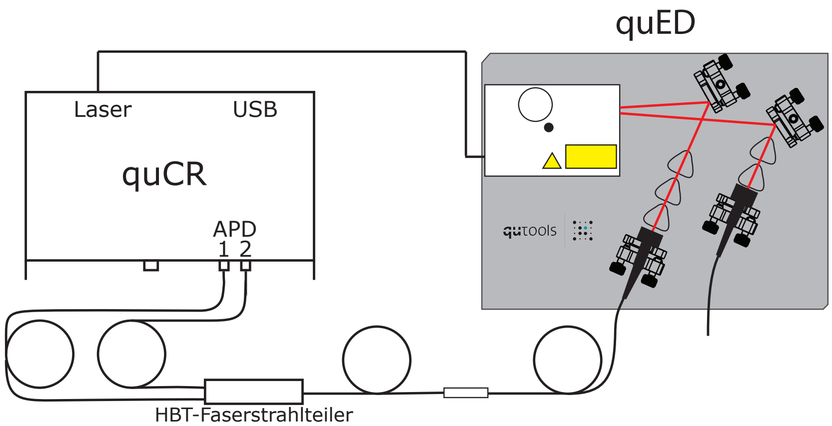

Figure 5: Schematic setup to answer the following question: Does the SPDC source output single, indivisible photons?

we switch off the pump laser, the counter is still counting events. They can come from stray light that hits the detectors, but even if we get rid of that, the detectors will still output pulses from time to time. They are called "dark counts".

2.2 Second Order Correlation Function for a Single Photon Source

So, we need something that lets us evaluate the measurement results in the real world with non-ideal equipment. How do we do this? You have already heard about this theoretically in the lecture on Monday. We are using the second order coherence function[5] $g^{(2)}(\tau = 0)$. Again, we will not derive it here, but we would like to motivate it in a very experimentalist way[6] We have already established that we are very interested in the 2-fold coincidences for this experiment. But instead of analysing only if they are 0 or not, we would like to find a measure that can live with slightly imperfect conditions. So, we would like to normalise these 2-fold coincidences between channel 1 and 2, N_{12}:

$$g^{(2)}(0) := \frac{N_{12}}{X} \tag{1}$$

To determine the normalising factor X, we can think of a rather boring case, where we had completely independent detections, possibly emitted from lots of different light sources. Whether two detection events of such a source would be registered within a certain coincidence time window would be completely random. There is nothing spacing detection events apart, but also nothing keeping them closer together, they are simply independent and that also makes them easy to handle.

Why could this be a good normalisation method? Well, if we had such a boring light source under examination, the $g^{(2)}(0)$ value would be 1. If we had a good single photon source, where you hardly ever detect coincidences, it would be lower than 1 (we speak of "anti-bunching" in this case). It the value would be larger than 1, there would be an increased probability of finding two detection events closer together than for independent ones (we call that "bunching").

I have convinced myself (and at least some of you, I hope) that this is a good way of doing it. But we still have to calculate it. So, we assume independent photons and independent detection events from two

[5] In general, the second order coherence function is time-dependent. This can be important, but in our case it is sufficient to look at one element of it, the $\tau = 0$ case.

[6] It is easy to do this, if you know what the theory says is correct.

Figure 6: The total measurement time is T and the time window width $\Delta_{tw} \ll T$. We assume that we can fit an integer number of these time windows in the total measurement time. There will be $\frac{T}{\Delta_{tw}}$ of these windows in T. We are trying to calculate the probabilities p_* that one or two detection events happen within one of these time windows.

detectors behind a beam splitter. We record and count single and coincidence events for a total time T. We have N_1 detection events in channel 1 (transmitted output of the BS) and N_2 events in channel 2 (reflected). Our coincidence time window has a width of Δ_{tw}. If you have 1 event, the probability of finding it in a specific a time slot of width Δ_{tw} is

$$\frac{\Delta_{tw}}{T} \quad , \tag{2}$$

because you have $\frac{T}{\Delta_{tw}}$ such time slots (see Fig. 6). When you have N_1 or N_2 events in total, the probability of finding a detection event in a time slot of width Δ_{tw} in channel 1 or 2 is

$$p_1 = N_1 \cdot \frac{\Delta_{tw}}{T} \qquad \text{and} \qquad p_2 = N_2 \cdot \frac{\Delta_{tw}}{T} \quad , \tag{3}$$

respectively. Since we are talking about **independent events**, it makes it very easy to express the joint probability, we simply have to multiply the two probabilities (the superscript a should emphasise the fact that we are looking at accidental coincidences):

$$p_{12}^a = p_1 \cdot p_2 = N_1 \cdot \frac{\Delta_{tw}}{T} \cdot N_2 \cdot \frac{\Delta_{tw}}{T} = N_1 N_2 \cdot \frac{\Delta_{tw}^2}{T^2} \tag{4}$$

This is the probability that a coincidence event happens in one of the coincidence time windows. If we add over the number of these time slots, we get what we wanted, the normalisation factor

$$X = \sum_{i=1}^{\frac{T}{\Delta_{tw}}} p_{12}^a = \sum_{i=1}^{\frac{T}{\Delta_{tw}}} N_1 N_2 \cdot \frac{\Delta_{tw}^2}{T^2} = \frac{T}{\Delta_{tw}} \cdot N_1 N_2 \cdot \frac{\Delta_{tw}^2}{T^2} = N_1 N_2 \cdot \frac{\Delta_{tw}}{T} \quad . \tag{5}$$

Side note: If you want to find out or check your coincidence time window Δ_{tw}, you can use this equation. If you have uncorrelated detection events (like from stray light), you can assume that $p_{12} = p_1 \cdot p_2$ and $N_{12} = N_1 N_2 \frac{\Delta_{tw}}{T}$. Solve for Δ_{tw}. We call N_{12} in the uncorrelated case **accidental coincidences**.

Back to the main issue: If we insert equation (5) into equation (1), we end up with:

$$g^{(2)}(0) = \frac{N_{12}}{N_1 N_2} \frac{T}{\Delta_{tw}} \tag{6}$$

And fortunately, this is exactly what the books say. With this progress in mind, let us re-evaluate our experiment from above. Again, we use the numbers from Fig. 3 and the additional information that our coincidence time window Δ_{tw} is 30 ns:

$$g^{(2)}(0) = \frac{27}{105500 \cdot 92033} \frac{10 \text{ s}}{30 \cdot 10^{-9} \text{ s}} = \frac{27}{9,709,481,500} \frac{10^9}{3} \approx 0.93 \tag{7}$$

Now, this is not close to 0, it is rather close to 1. In fact, if you take statistical (27 is a rather small number) and systematic (the coincidence time window is probably not exactly 30 ns) errors into account, our measurement will probably not exclude the 1.

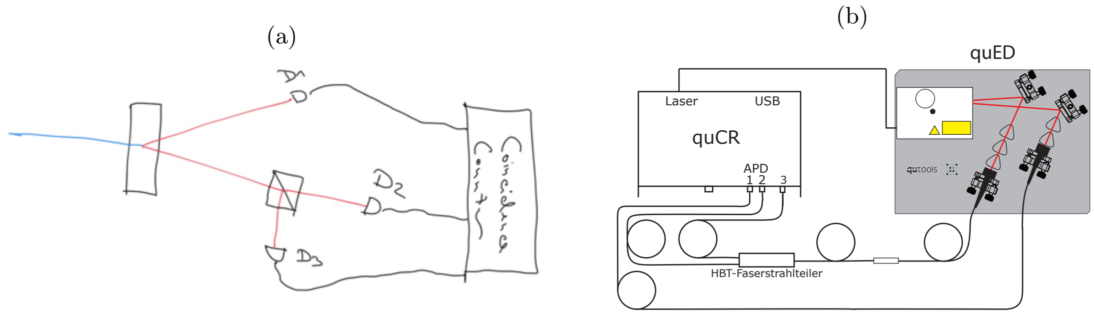

Figure 7: Setup to demonstrate heralded single photons. The very basic schematic on the left-hand side, how it is implemented with the quED on the right-hand side.

There could be other effects that do not permit us to show that this is a single photon source, but the truth is:

An SPDC source is NOT a single photon source.

Although this might sound disappointing at first, it is a very important message to take home.

2.3 Heralded Single Photon Source

Now we have not shown that photons are indivisible particles at all, but not all hope is lost. If you can't show that something is correct, it certainly does not mean that the opposite is true. We know that our source is not a single photon source[7]. I have already mentioned that the SPDC process has two output beams[8] and that energy is conserved. So, if the pump beam consists of photons of energy $\hbar\omega_p$, we will not be able to turn some of them into the same amount of photons with frequency $\omega_s = \omega_i$. But we could be able to generate a pair of photons from each pump photon we convert.

We can use this to our advantage. **If we detect a photon in one arm, we know that there is a very high probability[9] to have created exactly one photon in the other arm.** This is why we call an SPDC source a heralded single photon source. The detection of the photon in one arm heralds the existence of the one in the other arm.

2.4 Heralded Second Order Correlation Function

To show whether this hypothesis holds, we have to modify our experimental setup and the evaluation function. Since we are using one arm as the herald, we need to put our beam splitter in the other arm and we obviously need 3 detectors, now.

Obviously, the most important figure will be 3-fold coincidences between detector 1, 2 and 3 (N_{123}). With what we have learned before, when we do the experiment, we do not expect them to be 0 and the result is shown in Fig. 8. We know we need to normalise the 3-fold coincidences in a similar way as before, by calculating the heralded $g_H^{(2)}$ function. As normalisation we use the same reasoning as before with **independent events** (NOT what we expect to get from our source):

$$g_H^{(2)}(\tau = 0) := \frac{N_{123}}{Y} \tag{8}$$

[7] This statement includes the possibility that something like a photon does not exist.

[8] Three to be exact. Most of the pump beam is simply being transmitted through the crystal, but there is a filter in our setup that blocks it for safety reasons.

[9] There are limitations like dark counts, stray light, multiple pair emission etc.

The probability to find a 3-fold coincidence (number of single detection events N_1, N_2 and N_3 during measurement T in a coincidence time window Δ_{tw}) is accordingly (again, independent detection events, we use the superscript a for accidental to underline that for these probabilities):

$$p_{123}^a = p_1 \cdot p_2 \cdot p_3 = N_1 \cdot \frac{\Delta_{tw}}{T} \cdot N_2 \cdot \frac{\Delta_{tw}}{T} \cdot N_3 \cdot \frac{\Delta_{tw}}{T} = N_1 N_2 N_3 \cdot \frac{\Delta_{tw}^3}{T^3} \tag{9}$$

We could simply use this. However, since we get lots of coincidence counts from our counter, it seems like a more elegant idea to try and express p_{123}^a more in those:

$$p_{123}^a = p_{12}^a \cdot p_3 = p_{12}^a \cdot \frac{p_1}{p_1} \cdot p_3 = \frac{p_{12}^a \cdot p_{13}^a}{p_1} \tag{10}$$

With $N_{12} = \sum_1^{\frac{T}{\Delta_{tw}}} p_{12}^a = p_{12}^a \frac{T}{\Delta_{tw}}$, $N_{13} = \sum_1^{\frac{T}{\Delta_{tw}}} p_{13}^a = p_{13}^a \frac{T}{\Delta_{tw}}$ and $N_1 = \sum_1^{\frac{T}{\Delta_{tw}}} p_1 = p_1 \frac{T}{\Delta_{tw}}$ we get:

$$p_{123}^a = \frac{p_{12}^a \cdot p_{13}^a}{p_1} = \frac{N_{12}\frac{\Delta_{tw}}{T} N_{13}\frac{\Delta_{tw}}{T}}{N_1 \frac{\Delta_{tw}}{T}} = \frac{N_{12}N_{13}}{N_1} \cdot \frac{\Delta_{tw}}{T} \tag{11}$$

As above, we have to sum up the p_{123}^a over all time slots:

$$Y = \sum_{i=1}^{\frac{T}{\Delta_{tw}}} p_{123}^a = p_{123}^a \cdot \frac{T}{\Delta_{tw}} = \frac{N_{12}N_{13}}{N_1} \cdot \frac{\Delta_{tw}}{T} \cdot \frac{T}{\Delta_{tw}} = \frac{N_{12}N_{13}}{N_1} \tag{12}$$

And hence:

$$g_H^{(2)}(\tau = 0) = \frac{N_{123}}{Y} = \frac{N_{123}N_1}{N_{12}N_{13}} \tag{13}$$

Using the numbers from Fig. 8, we get:

$$g_H^{(2)}(0) = \frac{8 \cdot 234953}{11042 \cdot 11063} \approx 0.015 \tag{14}$$

This is a rather small number. Anything below 0.5 is considered a (heralded) single photon source. And it corroborates our thesis, that **our SPDC source emits pairs of indivisible units of light, i.e. photons**.

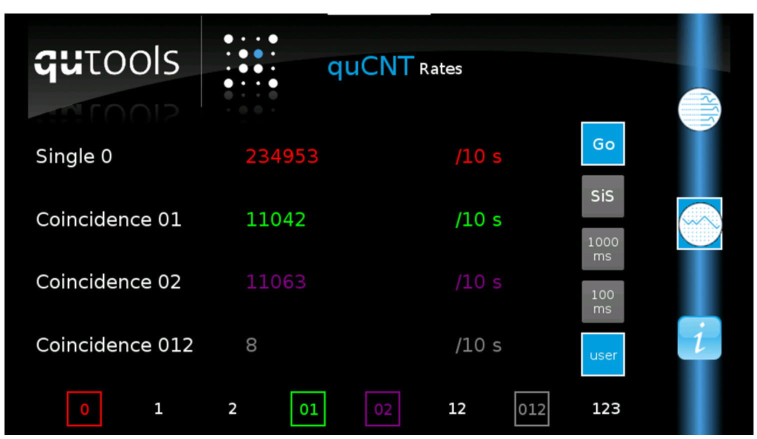

Figure 8: Screen shot of the quED performing the heralded single photon experiment. Only the necessary counts are shown.

2.5 Discussion of the Photon Pair Source

What makes an SPDC source a good heralded single photon source? We assume that our source converts pump photons into pairs of photons (with a very low efficiency, but that is not a problem, it can even be helpful). So, compared to the independent photon assumption, we now assume that with our source, there is a strong correlation between photons in the two arms (hence photon pairs). This will lead to the fact that the probability of detecting two photons does not decompose into the product of probabilities of finding each single (note that the superscript a is now missing, since the coincidences are not assumed accidental anymore):

$$p_{12} \gg p_1 \cdot p_2 \qquad \text{and} \qquad p_{13} \gg p_1 \cdot p_3 \tag{15}$$

Because of the way the probabilities translate to the number of events during the measurement time, i.e. $N_* = p_* \cdot \frac{T}{\Delta_{tw}}$, it is easy to see that the second order correlation in the heralded case can be expressed in the same way using probabilities (which is not true for the non-heralded case):

$$g_H^{(2)}(\tau = 0) = \frac{N_{123} N_1}{N_{12} N_{13}} = \frac{p_{123} p_1}{p_{12} p_{13}} \tag{16}$$

Now, p_{12} and p_{13} do not decompose into the products and neither does p_{123}, but the latter can be decomposed, if we assume that we basically have coincidences in either 1 and 2 or 1 and 3 and a spurious background event or double pair event that results in an additional detection in the third detector:

$$p_{123} = p_{12} \cdot p_3 + p_{13} \cdot p_2 \tag{17}$$

This leads to:

$$g_H^{(2)}(\tau = 0) = \frac{(p_{12} p_3 + p_{13} p_2) p_1}{p_{12} p_{13}} \tag{18}$$

If we further assume that the probabilities of getting coincidences between 1 and 2 and 1 and 3 are pretty much equal ($p_{12} \approx p_{13}$) and singles in 2 and 3 ($p_2 \approx p_3$), too, because of similar detection efficiencies and a 50:50 beam splitter, we can reduce that to:

$$g_H^{(2)}(\tau = 0) \approx \frac{(p_{12} p_2 + p_{12} p_2) p_1}{p_{12} p_{12}} = \frac{2 p_{12} p_2 p_1}{p_{12} p_{12}} = \frac{2 p_2 p_1}{p_{12}} \tag{19}$$

Since one of our assumptions was that $p_{12} \gg p_1 p_2$ it seems clear, why we get the low $g_H^{(2)}(0)$ value. The strong correlations between detectors 1 and 3 and 2 and 3, respectively, make the difference.

2.6 Some Additional Facts and Background

- Although our SPDC source is not a single photon source (without heralding), there are such physical systems, [4–6] amongst many others. However, since photon pair sources have been around rather early, they were among the first to show the anti-correlation effect [7].
- The setup of correlating the detection time events of two detectors has been invented already in the 1950s, by Robert Hanbury Brown and Richard Q. Twiss [8]. Their application, however, seems very different at first glance: They wanted to measure the radius of stars and used to optical telescopes with photomultiplier tubes (a different kind of single photon detectors) attached. The time correlation of their output pulses was analysed. And although the phase information of the incoming light is lost in the detection process, this setup (and what we have used above) is called a Hanbury Brown & Twiss interferometer.
- The “independent” events we have been comparing everything against are being produced by a so-called “coherent state”. An ideal laser produces such a state.

References

[1] Mark Beck. *Quantum mechanics : theory and experiment*. Oxford University Press, New York, 2012. ISBN 9780199798230 0199798230 1280881666 9781280881664. URL `http://site.ebrary.com/id/10578493`.

[2] Gilbert Grynberg, Alain Aspect, Claude Fabre, and Claude Cohen-Tannoudji. *Introduction to quantum optics from the semi-classical approach to quantized light*. Cambridge University Press, 2012. ISBN 9780511778261 0511778260. URL `https://doi.org/10.1017/CBO9780511778261`.

[3] Paul G. Kwiat, Edo Waks, Andrew G. White, Ian Appelbaum, and Philippe H. Eberhard. Ultrabright source of polarization-entangled photons. *Physical Review A*, 60(2):R773–R776, 08 1999. doi: 10.1103/PhysRevA.60.R773. URL `https://link.aps.org/doi/10.1103/PhysRevA.60.R773`.

[4] A. Imamoglu and Y. Yamamoto. Turnstile device for heralded single photons: Coulomb blockade of electron and hole tunneling in quantum confined p-i-n heterojunctions. *Phys. Rev. Lett.*, 72:210–213, Jan 1994. doi: 10.1103/PhysRevLett.72.210. URL `https://link.aps.org/doi/10.1103/PhysRevLett.72.210`.

[5] P. Michler, A. Kiraz, C. Becher, W. V. Schoenfeld, P. M. Petroff, Lidong Zhang, E. Hu, and A. Imamoglu. A quantum dot single-photon turnstile device. *Science*, 290(5500):2282–2285, 2000. ISSN 0036-8075. doi: 10.1126/science.290.5500.2282. URL `https://science.sciencemag.org/content/290/5500/2282`.

[6] Frank Diedrich and Herbert Walther. Nonclassical radiation of a single stored ion. *Phys. Rev. Lett.*, 58:203–206, Jan 1987. doi: 10.1103/PhysRevLett.58.203. URL `https://link.aps.org/doi/10.1103/PhysRevLett.58.203`.

[7] P Grangier, G Roger, and A Aspect. Experimental evidence for a photon anticorrelation effect on a beam splitter: A new light on single-photon interferences. *Europhysics Letters (EPL)*, 1(4):173–179, feb 1986. doi: 10.1209/0295-5075/1/4/004. URL `https://doi.org/10.1209%2F0295-5075%2F1%2F4%2F004`.

[8] R. Hanbury Brown and R. Q. Twiss. Correlation between photons in two coherent beams of light. *Nature*, 177(4497):27–29, 1956. doi: 10.1038/177027a0. URL `https://doi.org/10.1038/177027a0`.

Acronyms

BBO β-Barium Borate. 1,

BS Beam Splitter. 1, 3, 5,

SPAD Single Photon Avalanche Detector. 1–3,

SPDC Spontaneous Parametric Down-Conversion. 1–4, 6–8